Analysis of Inductive WPT

Jan Kracek, Milos Mazanek

Czech Technical University in Prague

WIPE

COST IC1301

European Science Foundation

COST

EUROPEAN COOPERATION IN SCIENCE AND TECHNOLOGY

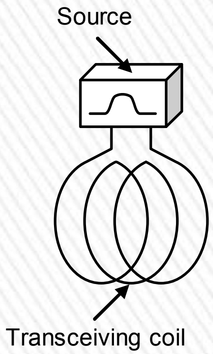
Outline

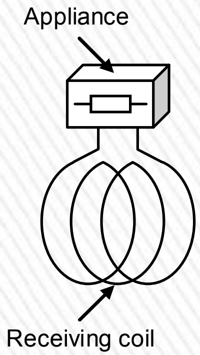
» Introduction
» Efficiency
» Power balance
» Input impedance
» Conclusion

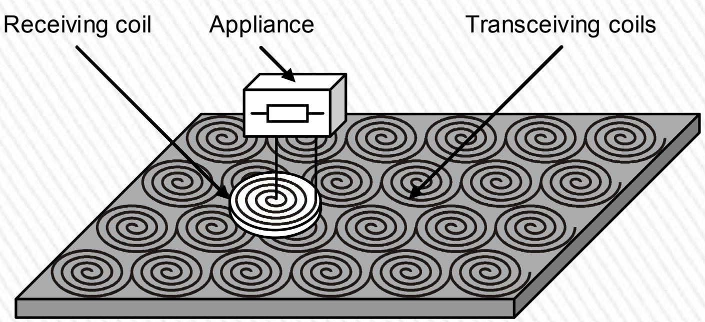
Outline

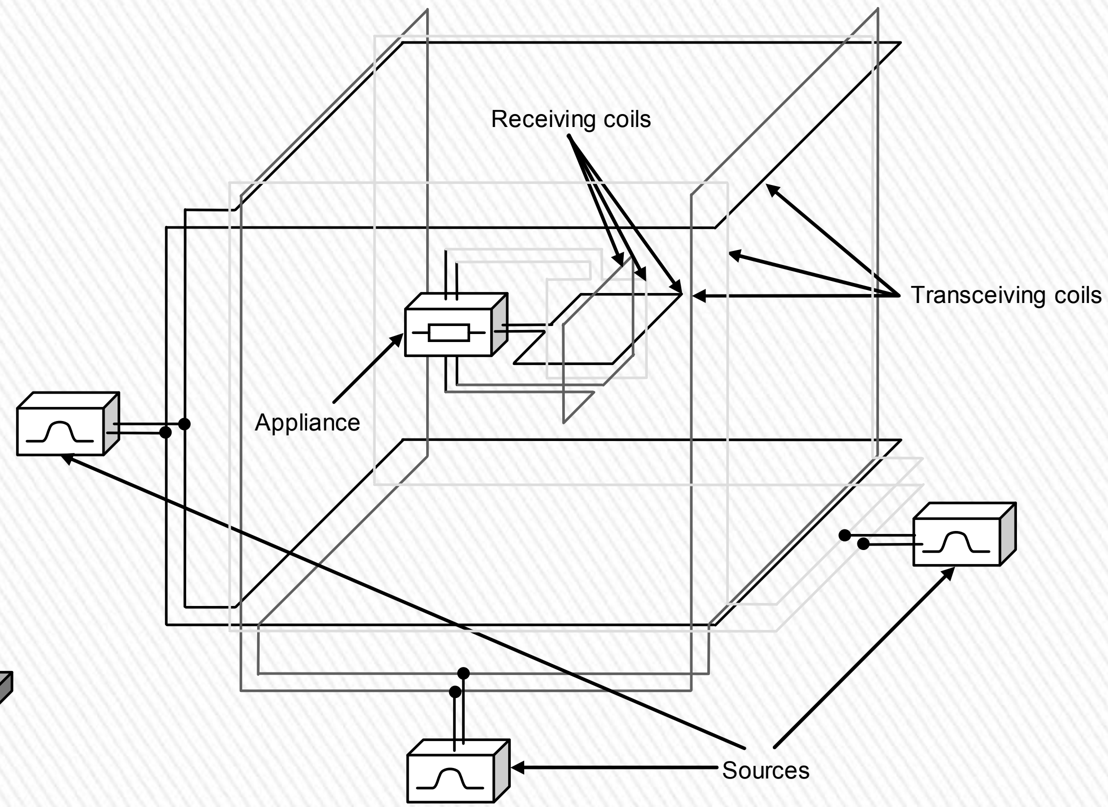
- » Introduction
- » Efficiency
- » Power balance
- » Input impedance
- » Conclusion

Introduction

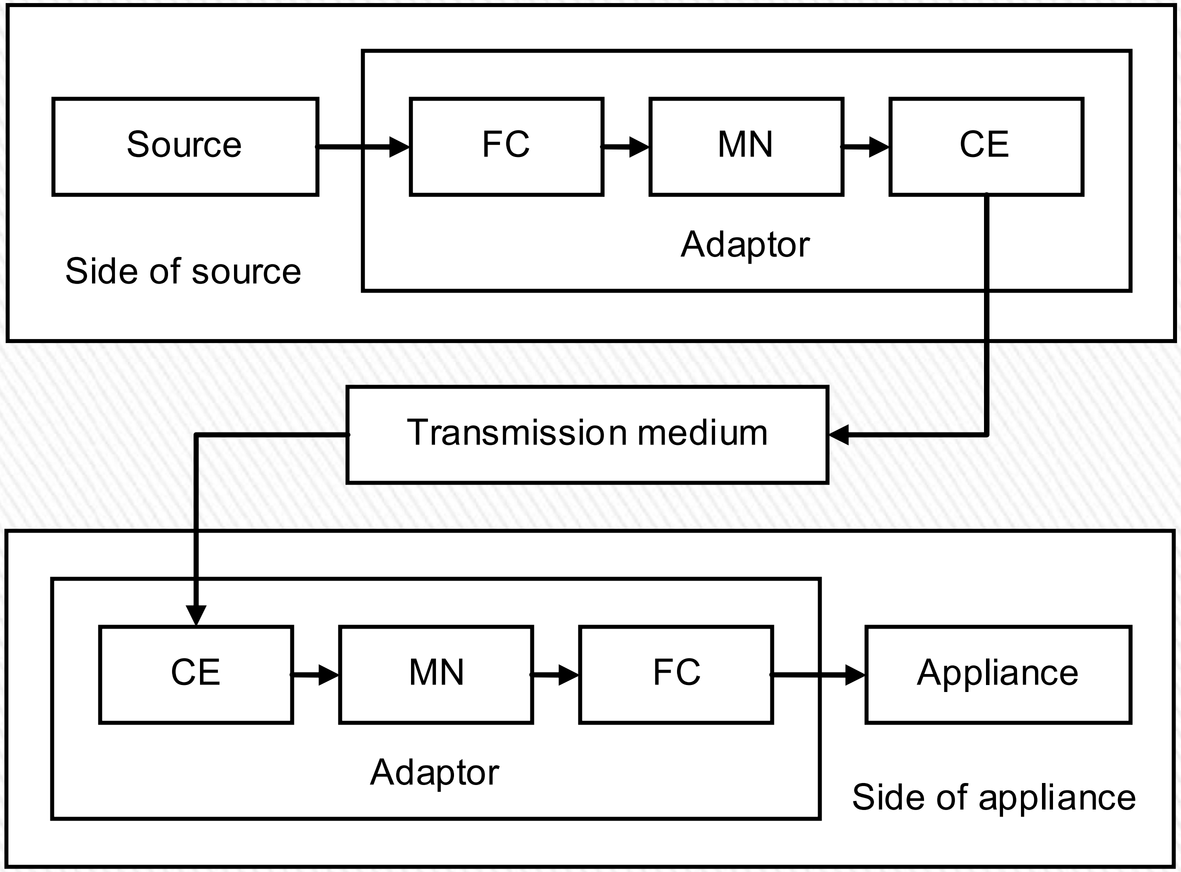




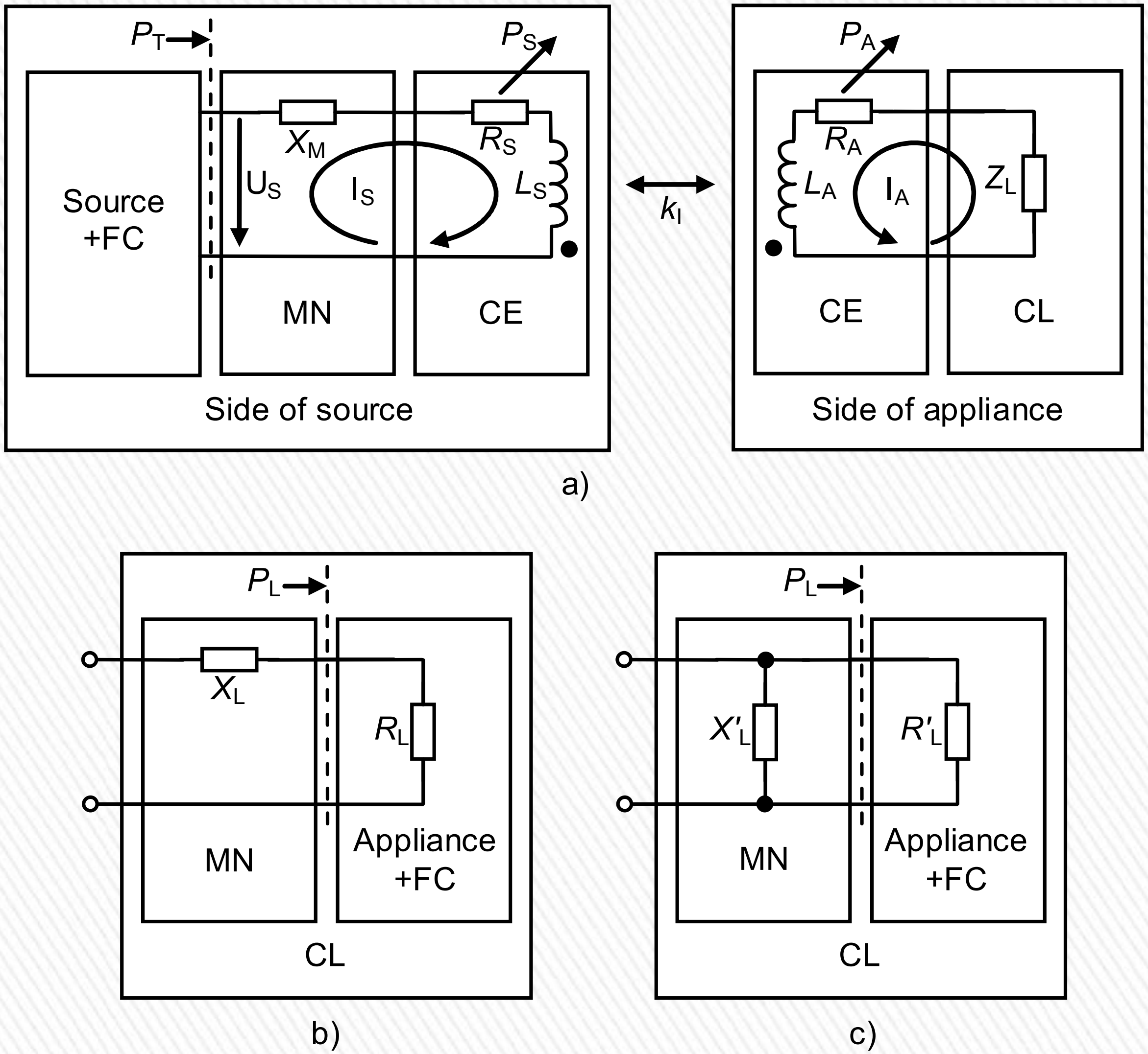




General WPT Chain



Circuit Model



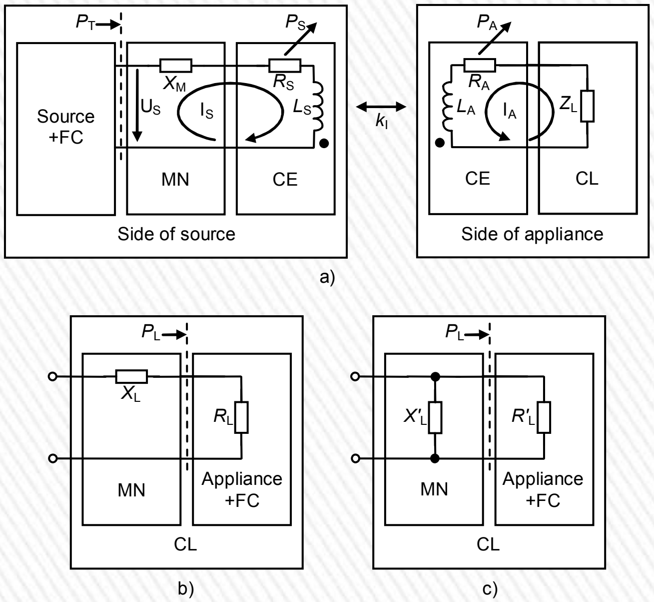
Outline

- » Introduction
- » Efficiency
- » Power balance
- » Input impedance
- » Conclusion

Efficiency Definition

» Main losses are caused by resistances of transceiving and receiving coils.

» $\eta_{\mathrm{I}} = \dfrac{P_{\mathrm{L}}}{P_{\mathrm{T}}} = \dfrac{P_{\mathrm{L}}}{P_{\mathrm{A}} + P_{\mathrm{L}} + P_{\mathrm{S}}}$



Efficiency Derivation (1)

» $$\eta_I = \frac{R_L X_K^2}{\left(\left(R_A + R_L\right)^2 + \left(X_A + X_L\right)^2\right) R_S + \left(R_A + R_L\right) X_K^2}$$

$$X_A = \omega L_A,\ X_S = \omega L_S,\ X_K = \omega k_I \sqrt{L_A L_S}$$

$$\frac{X_K}{\sqrt{X_A X_S}} = k_I,\ \frac{X_A}{R_A} = Q_A,\ \frac{X_L}{R_A} = -Q'_A,\ \frac{X_A}{R_L} = Q_L,\ \frac{X_S}{R_S} = Q_S$$

» $$\eta_I = \frac{k_I^2 Q_A Q_S \dfrac{Q_A}{Q_L}}{\left(1 + \dfrac{Q_A}{Q_L}\right)^2 + \left(Q_A - Q'_A\right)^2 + k_I^2 Q_A Q_S \left(1 + \dfrac{Q_A}{Q_L}\right)}$$

Efficiency Derivation (2)

» $\kappa = k_\mathrm{I}\sqrt{Q_\mathrm{A} Q_\mathrm{S}}$, $\rho = \dfrac{Q_\mathrm{A}}{Q_\mathrm{L}}$, $\xi = Q_\mathrm{A} - Q'_\mathrm{A}$

$$\eta_\mathrm{I} = \frac{\kappa^2 \rho}{(1+\rho)^2 + \xi^2 + \kappa^2 (1+\rho)}$$

» R, L, C, ω, (X) $\rightarrow$ k_I, Q_A, Q'_A, Q_L, Q_S $\rightarrow$ κ, ρ, ξ $\rightarrow$

$\max\left(\eta_\mathrm{I}(\rho, \xi)\right) \rightarrow \rho = \sqrt{1+\kappa^2}$, $\xi = 0 \rightarrow$

κ, ρ, ξ $\rightarrow$ k_I, Q_A, Q'_A, Q_L, Q_S $\rightarrow$ R, L, C, ω, (X)

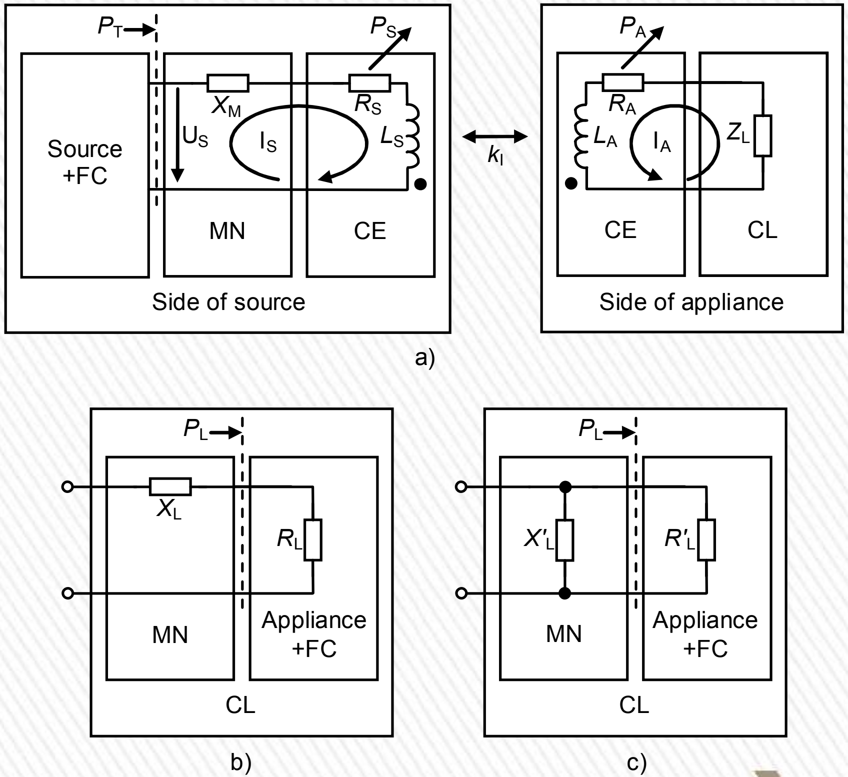
Maximal Efficiency, Optimal Conditions

» $$R_L = \sqrt{R_A^2 + \omega^2 k_l^2 L_A L_S \frac{R_A}{R_S}}$$

$$X_L = -\omega L_A$$

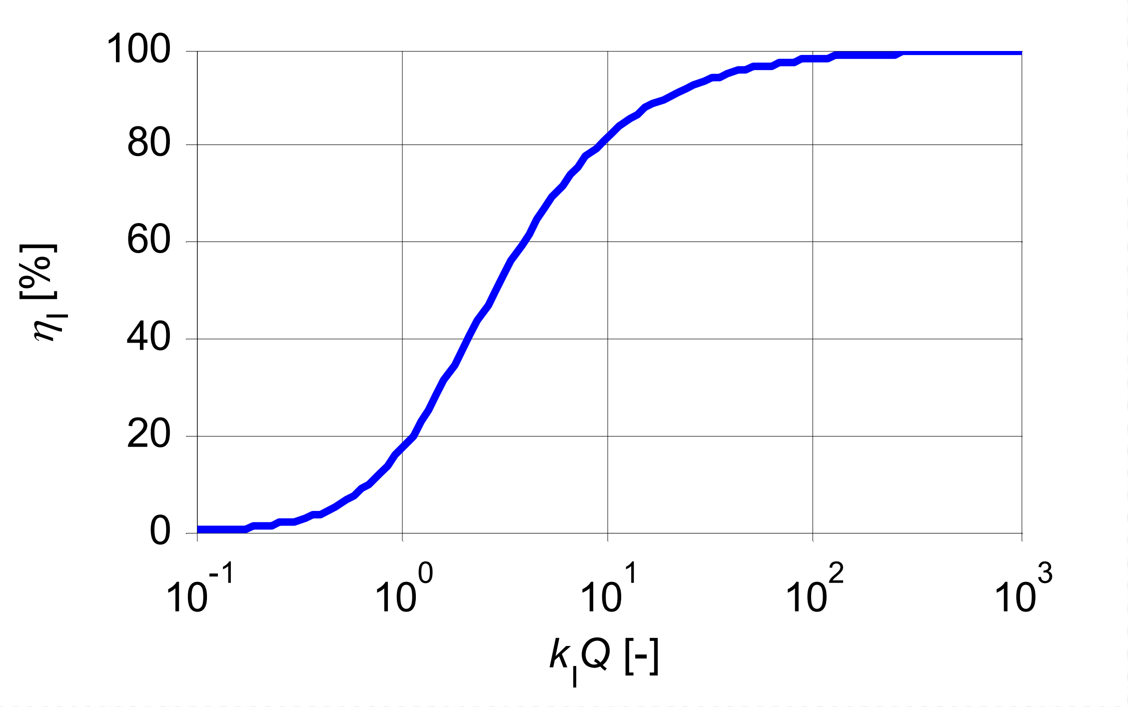
» $$\eta_l = \frac{(k_l Q)^2}{\left(1 + \sqrt{1 + (k_l Q)^2}\right)^2}$$

$$Q = \sqrt{Q_A Q_S}$$



Maximal Efficiency

» $$\eta_l = \frac{\left(k_l Q\right)^2}{\left(1+\sqrt{1+\left(k_l Q\right)^2}\right)^2}$$

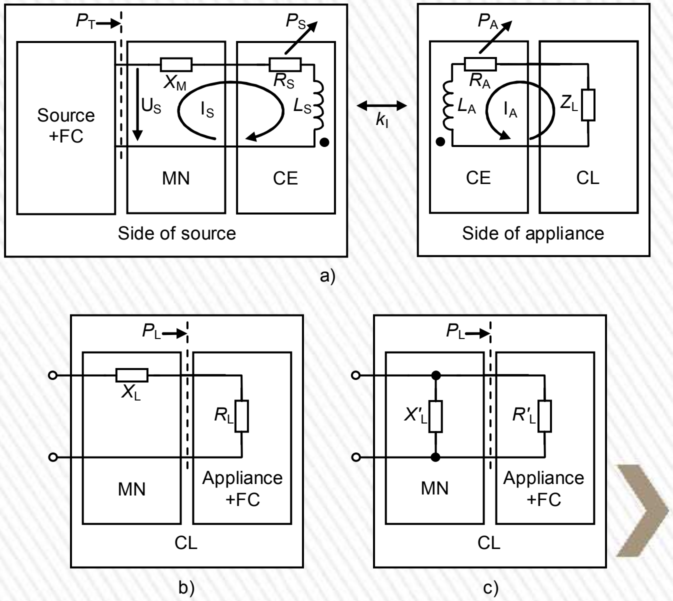


Max. eff., Opt. Con. – Dual Case

» $$R_L + jX_L = \frac{R'_L X'^2_L}{R'^2_L + X'^2_L} + j\frac{R'^2_L X'_L}{R'^2_L + X'^2_L}$$

» $$R'_L = \frac{\omega^2 L_A^2}{\sqrt{R_A^2 + \omega^2 k_I^2 L_A L_S \frac{R_A}{R_S}}} + \sqrt{R_A^2 + \omega^2 k_I^2 L_A L_S \frac{R_A}{R_S}}$$

$$X'_L = -\omega L_A - \frac{R_A^2 + \omega^2 k_M^2 L_A L_S \frac{R_A}{R_S}}{\omega L_A}$$

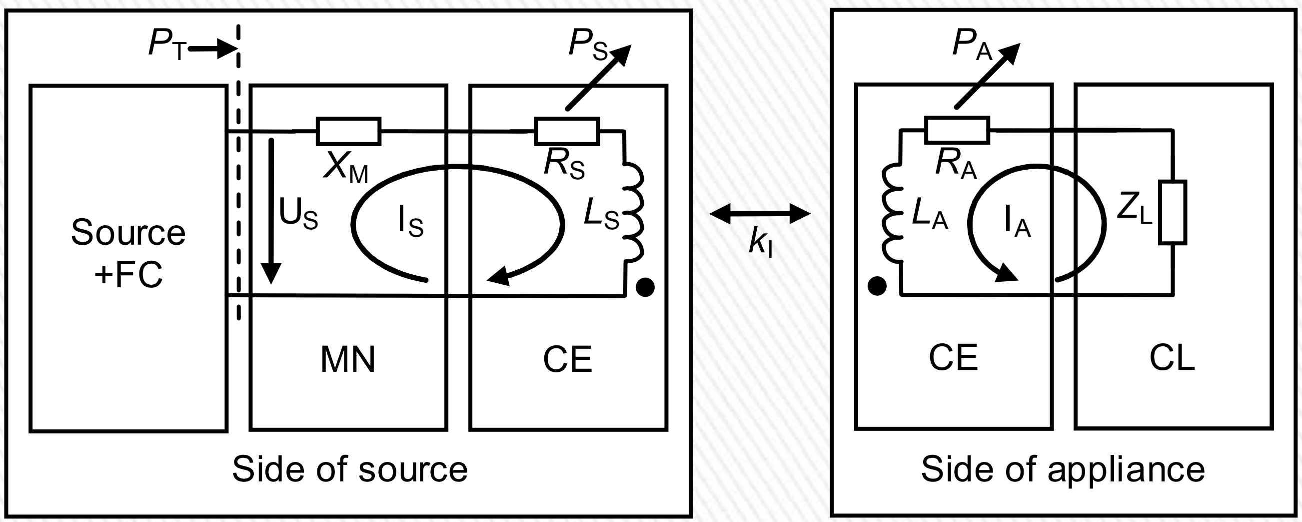


Outline

- Introduction
- Efficiency
- Power balance
- Input impedance
- Conclusion

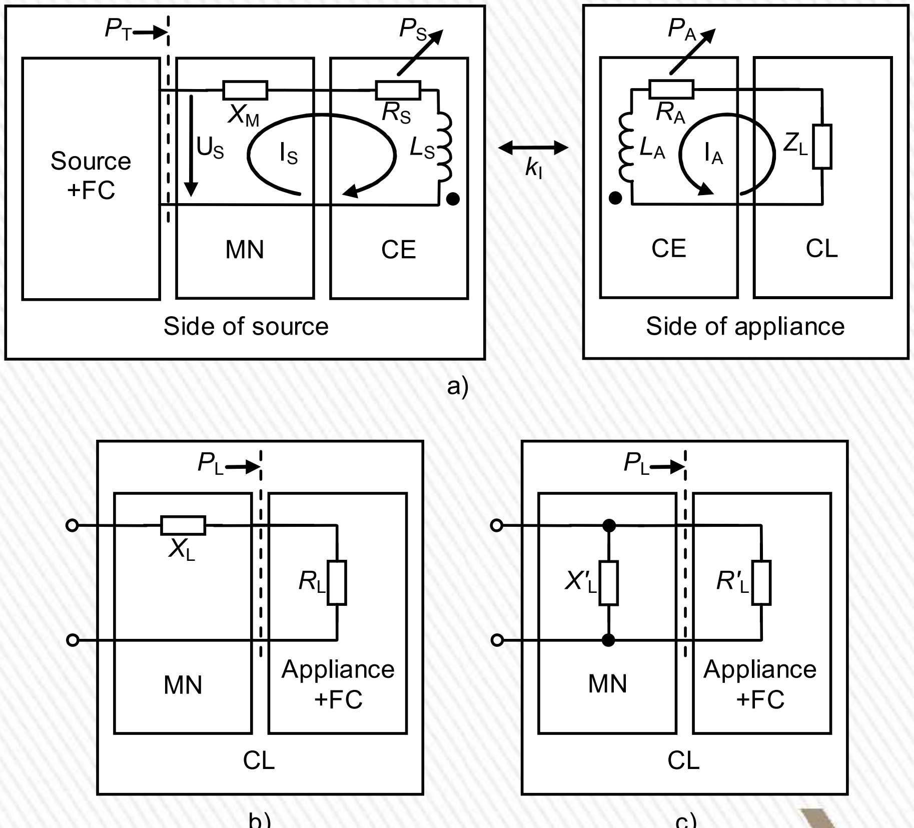
Maximal Appliance Power, Optimal Condition

- $\max\left(P_L\left(X_M\right)\right)$
- $X_M = -\omega L_S$



Power Balance (1)

- $$R_L = \sqrt{R_A^2 + \omega^2 k_I^2 L_A L_S \frac{R_A}{R_S}}$$
- $$X_L = -\omega L_A$$
- $$X_M = -\omega L_S$$



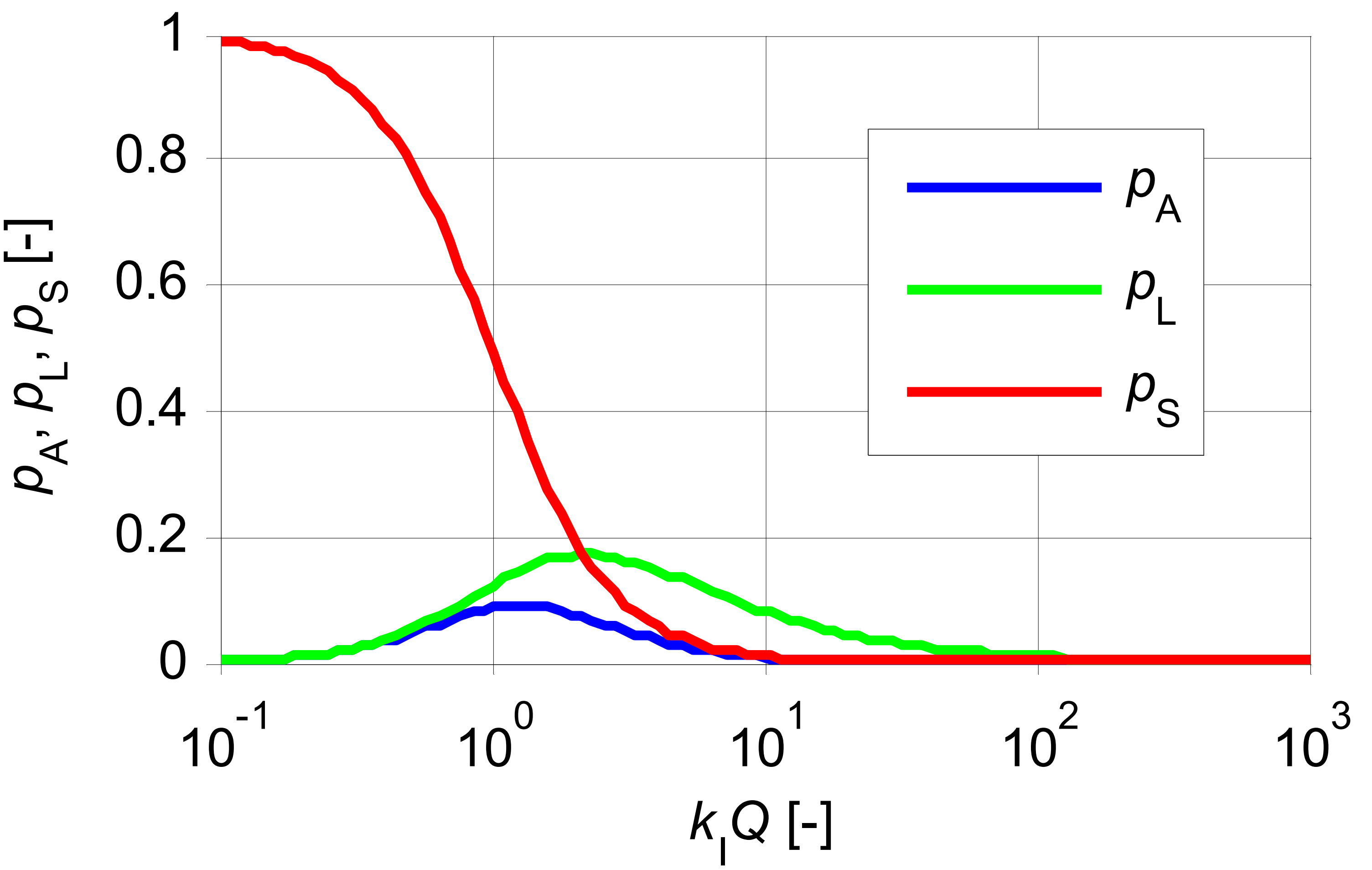
Power Balance (2)

- $$P_{\mathrm{A}} = \underbrace{\frac{\left(k_{\mathrm{l}}Q\right)^2}{\left(1+\left(k_{\mathrm{l}}Q\right)^2+\sqrt{1+\left(k_{\mathrm{l}}Q\right)^2}\right)^2}}_{p_{\mathrm{A}}}\frac{\left|\mathrm{U}_{\mathrm{S}}\right|^2}{R_{\mathrm{S}}}$$

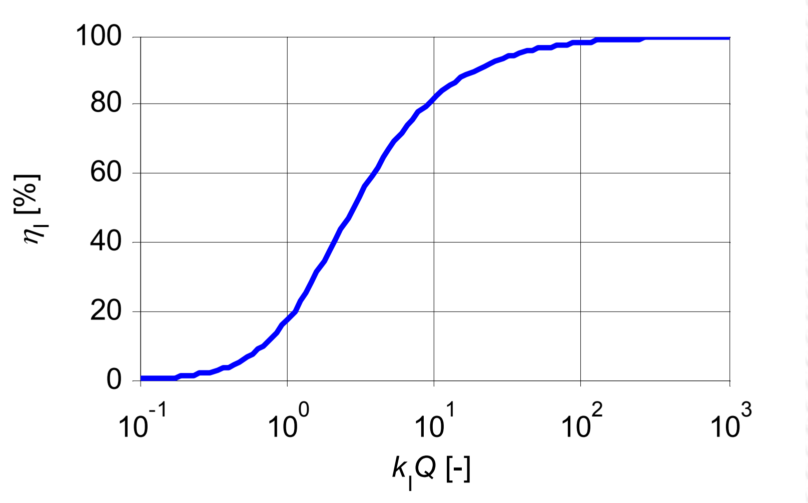
- $$P_{\mathrm{L}} = \underbrace{\frac{\left(k_{\mathrm{l}}Q\right)^2\sqrt{1+\left(k_{\mathrm{l}}Q\right)^2}}{\left(1+\left(k_{\mathrm{l}}Q\right)^2+\sqrt{1+\left(k_{\mathrm{l}}Q\right)^2}\right)^2}}_{p_{\mathrm{L}}}\frac{\left|\mathrm{U}_{\mathrm{S}}\right|^2}{R_{\mathrm{S}}}$$

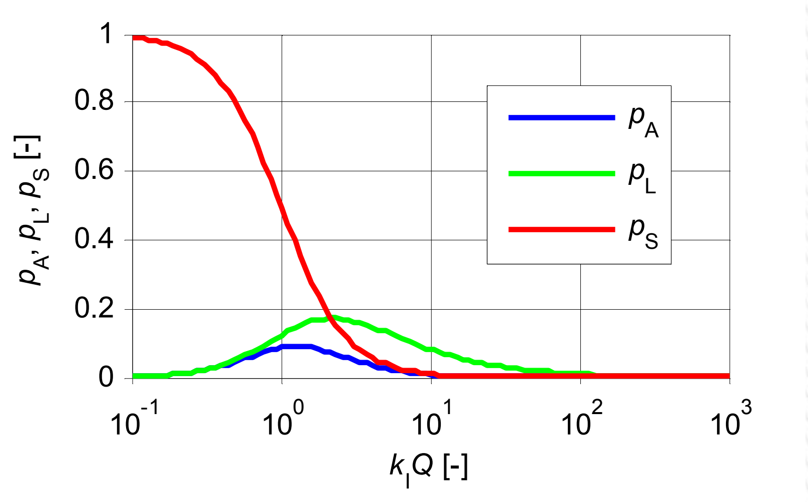
- $$P_{\mathrm{S}} = \underbrace{\frac{\left(1+\sqrt{1+\left(k_{\mathrm{l}}Q\right)^2}\right)^2}{\left(1+\left(k_{\mathrm{l}}Q\right)^2+\sqrt{1+\left(k_{\mathrm{l}}Q\right)^2}\right)^2}}_{p_{\mathrm{S}}}\frac{\left|\mathrm{U}_{\mathrm{S}}\right|^2}{R_{\mathrm{S}}}$$

Power Balance (3)



Efficiency and Power Balance



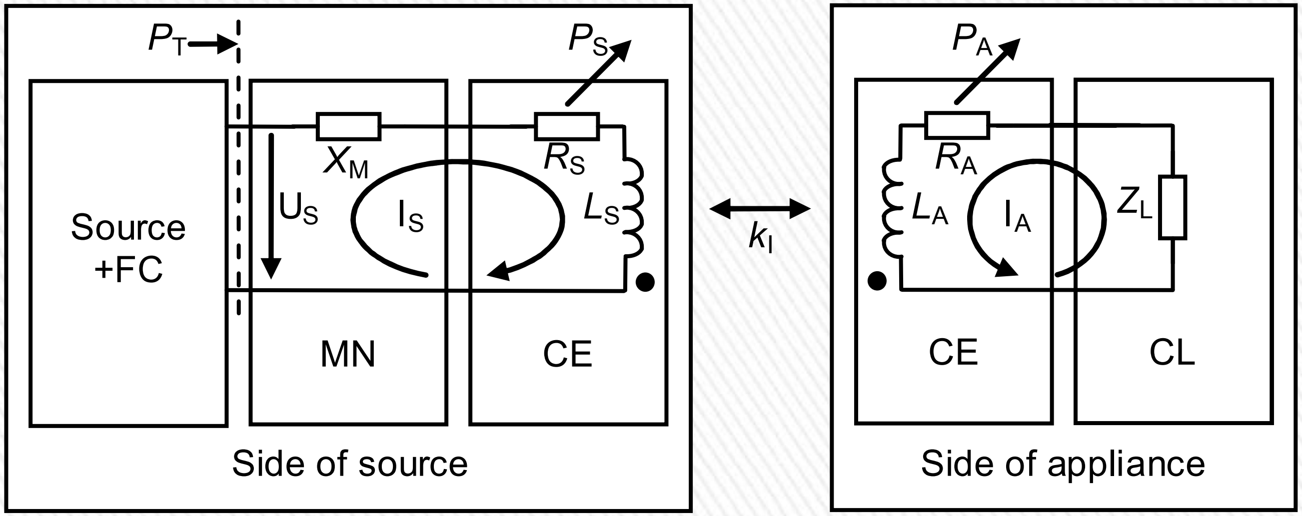


Outline

» Introduction
» Efficiency
» Power balance
» Input impedance
» Conclusion

Input Impedance (1)

» $Z_S = \frac{U_S}{I_S}$



Input Impedance (2)

» $$Z_S = \frac{U_S}{I_S} = R_S + \frac{\left(R_A + R_L\right) X_K^2}{\left(R_A + R_L\right)^2 + \left(X_A + X_L\right)^2} + $$

$$+ j\left(X_M + X_S - \frac{\left(X_A + X_L\right) X_K^2}{\left(R_A + R_L\right)^2 + \left(X_A + X_L\right)^2} \right)$$

» By optimal conditions for efficiency and appliance power:

$$Z_S = R_S + \frac{\omega^2 k_I^2 L_A L_S}{R_A + \sqrt{R_A^2 + \omega^2 k_I^2 L_A L_S \frac{R_A}{R_S}}}$$

Outline

» Introduction
» Efficiency
» Power balance
» Input impedance
» Conclusion

Conclusion

- The general circuit model for WPT by electromagnetic induction was shown.
- The efficiency was mentioned.
- The power balance in the terms of normalized powers was derived.
- The input impedance was presented.